

Exercice 2.2.1

Résous les équations ci-dessous.

On précisera dans chaque cas la valeur des trois coefficients a , b et c ainsi que la valeur du discriminant $\Delta = b^2 - 4ac$. Donne les solutions en valeur exacte (avec les racines) et en valeur numérique arrondie à deux décimales.

a) $9x^2 - 6x + 1 = 0$

$$a = 9 \quad b = -6 \quad c = 1$$

$$\Delta = b^2 - 4ac = 36 - 36 = 0$$

$$x = \frac{6}{18} = \frac{1}{3}$$

$$S = \left\{ \frac{1}{3} \right\}$$

b) $3x^2 - 4x + 1 = 0$

$$a = 3 \quad b = -4 \quad c = 1$$

$$\Delta = b^2 - 4ac = 16 - 12 = 4$$

$$x = \frac{4 \pm \sqrt{4}}{6} = \frac{4 \pm 2}{6}$$

$$x_1 = \frac{4+2}{6} = 1 \quad x_2 = \frac{4-2}{6} = \frac{1}{3}$$

$$S = \left\{ \frac{1}{3}; 1 \right\}$$

c) $4x^2 + 4x + 1 = 0$

$$a = 4 \quad b = 4 \quad c = 1$$

$$\Delta = b^2 - 4ac = 16 - 16 = 0$$

$$x = -\frac{4}{8} = -\frac{1}{2}$$

$$S = \left\{ -\frac{1}{2} \right\}$$

d) $4x^2 + 6x + 3 = 0$

$$a = 4 \quad b = 6 \quad c = 3$$

$$\Delta = b^2 - 4ac = 36 - 48 = -12$$

$$S = \emptyset$$

e) $25x^2 - 10x + 1 = 0$

$$a = 25 \quad b = -10 \quad c = 1$$

$$\Delta = b^2 - 4ac = 100 - 100 = 0$$

$$x = \frac{10}{50} = \frac{1}{5}$$

$$S = \left\{ \frac{1}{5} \right\}$$

Exercice 2.2.2

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $4x^2 + 12x + 9 = 0$

$$a = 4 \quad b = 12 \quad c = 9$$

$$\Delta = b^2 - 4ac = 144 - 144 = 0$$

$$x = -\frac{12}{8} = -\frac{3}{2}$$

$$S = \left\{-\frac{3}{2}\right\}$$

b) $16x^2 + 8x + 1 = 0$

$$a = 16 \quad b = 8 \quad c = 1$$

$$\Delta = b^2 - 4ac = 64 - 64 = 0$$

$$x = -\frac{8}{32} = -\frac{1}{4}$$

$$S = \left\{-\frac{1}{4}\right\}$$

c) $2x^2 + 6x + 3 = 0$

$$a = 2 \quad b = 6 \quad c = 3$$

$$\Delta = b^2 - 4ac = 36 - 24 = 12$$

$$x_1 = \frac{-6 + \sqrt{12}}{4} = \frac{-6 + 2\sqrt{3}}{4} = \frac{-3 + \sqrt{3}}{2}$$

$$x_2 = \frac{-3 - \sqrt{3}}{2}$$

$$S = \left\{\frac{-3 - \sqrt{3}}{2}; \frac{-3 + \sqrt{3}}{2}\right\}$$

d) $-3x^2 + 8x + 4 = 0$

$$a = -3 \quad b = 8 \quad c = 4$$

$$\Delta = b^2 - 4ac = 64 + 48 = 112$$

$$x_1 = \frac{-8 + \sqrt{112}}{-6} = \frac{8 + 4\sqrt{7}}{6} = \frac{4 + 2\sqrt{7}}{3}$$

$$x_2 = \frac{4 - 2\sqrt{7}}{3}$$

$$S = \left\{\frac{4 - 2\sqrt{7}}{3}; \frac{4 + 2\sqrt{7}}{3}\right\}$$

Exercice 2.2.2

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $x^2 - 3x + 8 = 0$

$$a = 1 \quad b = -3 \quad c = 8$$

$$\Delta = b^2 - 4ac = 9 - 32 = -23$$

$$S = \emptyset$$

b) $5x^2 - 10x - 7 = 0$

$$a = 5 \quad b = -10 \quad c = -7$$

$$\Delta = b^2 - 4ac = 100 + 140 = 240$$

$$x_1 = \frac{10 + \sqrt{240}}{10} = \frac{10 + 4\sqrt{15}}{10} = \frac{5 + 2\sqrt{15}}{5}$$

$$x_2 = \frac{5 - 2\sqrt{15}}{5}$$

$$S = \left\{ \frac{5 - 2\sqrt{15}}{5}; \frac{5 + 2\sqrt{15}}{5} \right\}$$

c) $4x^2 + 16x + 16 = 0$

$$a = 4 \quad b = 16 \quad c = 16$$

$$\Delta = b^2 - 4ac = 256 - 256 = 0$$

$$x = -\frac{16}{8} = -2$$

$$S = \{-2\}$$

ou

$$4(x^2 + 4x + 4) = 0$$

$$4(x + 2)^2 = 0$$

$$x + 2 = 0 \quad x = -2$$

$$S = \{-2\}$$

d) $x^2 - 4x + 8 = 0$

$$a = 1 \quad b = -4 \quad c = 8$$

$$\Delta = b^2 - 4ac = 16 - 32 = -16$$

$$S = \emptyset$$

e) $x^2 + 6x - 9 = 0$

$$a = 1 \quad b = 6 \quad c = -9$$

$$\Delta = b^2 - 4ac = 36 + 36 = 72$$

$$x_1 = \frac{-6 + \sqrt{72}}{2} = \frac{-6 + 6\sqrt{2}}{2} = -3 + 3\sqrt{2}$$

$$x_2 = -3 - 3\sqrt{2}$$

$$S = \{-3 - 3\sqrt{2}; -3 + 3\sqrt{2}\}$$

Exercice 2.2.3

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $3x^2 - 30x + 74 = 0$

$$a = 3 \quad b = -30 \quad c = 74$$

$$\Delta = b^2 - 4ac = 900 - 888 = 12$$

$$x_1 = \frac{30 + \sqrt{12}}{6} = \frac{30 + 2\sqrt{3}}{6} = \frac{15 + \sqrt{3}}{3}$$

$$x_2 = \frac{15 - \sqrt{3}}{3}$$

$$S = \left\{ \frac{15 - \sqrt{3}}{3}; \frac{15 + \sqrt{3}}{3} \right\}$$

b) $-3x^2 - 30x - 79 = 0$

$$a = -3 \quad b = -30 \quad c = -79$$

$$\Delta = b^2 - 4ac = 900 - 948 = -48$$

$$S = \emptyset$$

c) $-3x^2 + 6x - 4 = 0$

$$a = -3 \quad b = 6 \quad c = -4$$

$$\Delta = b^2 - 4ac = 36 - 48 = -12$$

$$S = \emptyset$$

d) $4x^2 + 24x + 36 = 0$

$$x^2 + 6x + 9 = 0$$

$$(x + 3)^2 = 0$$

$$x + 3 = 0$$

$$S = \{-3\}$$

e) $9x^2 + 6x + 5 = 0$

$$\Delta = b^2 - 4ac = 36 - 180 = -144$$

$$S = \emptyset$$

f) $-x^2 - 8x - 15 = 0$

$$x^2 + 8x + 15 = 0$$

$$(x + 5)(x + 3) = 0$$

$$x + 5 = 0 \quad x_1 = -5$$

$$x + 3 = 0 \quad x_2 = -3$$

$$S = \{-5; -3\}$$

Exercice 2.2.4

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $-x^2 - 6x - 10 = 0$

$$x^2 + 6x + 10 = 0$$

$$\Delta = b^2 - 4ac = 36 - 40 = -4$$

$$S = \emptyset$$

b) $3x^2 - 6x + 3 = 0$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x - 1 = 0 \quad x = 1$$

$$S = \{1\}$$

c) $2x^2 - 12x + 22 = 0$

$$x^2 - 6x + 11 = 0$$

$$\Delta = b^2 - 4ac = 36 - 44 = -8$$

$$S = \emptyset$$

d) $4x^2 + 16x - 16 = 0$

$$x^2 + 4x - 4 = 0$$

$$\Delta = b^2 - 4ac = 16 + 16 = 32$$

$$x = \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 4\sqrt{2}}{2}$$

$$x_1 = \frac{-4 + 4\sqrt{2}}{2} = -2 + 2\sqrt{2} \quad x_2 = \frac{-4 - 4\sqrt{2}}{2} = -2 - 2\sqrt{2}$$

$$S = \{-2 - 2\sqrt{2}; -2 + 2\sqrt{2}\}$$

e) $2x^2 - 16x + 37 = 0$

$$\Delta = b^2 - 4ac = 256 - 296 = -40$$

$$S = \emptyset$$

f) $3x^2 - 6x + 8 = 0$

$$\Delta = b^2 - 4ac = 36 - 96 = -60$$

$$S = \emptyset$$

Exercice 2.2.5

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $2x^2 + 16x - 32 = 0$

$$x^2 + 8x - 16 = 0$$

$$\Delta = b^2 - 4ac = 64 + 64 = 128$$

$$x = \frac{-8 \pm \sqrt{128}}{2} = \frac{-8 \pm 8\sqrt{2}}{2}$$

$$x_1 = \frac{-8 + 8\sqrt{2}}{2} = -4 + 4\sqrt{2} \quad x_2 = \frac{-8 - 8\sqrt{2}}{2} = -4 - 4\sqrt{2}$$

$$S = \{-4 - 4\sqrt{2}; -4 + 4\sqrt{2}\}$$

b) $3x^2 - 6x + 3 = 0$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x - 1 = 0 \quad x = 1$$

$$S = \{1\}$$

c) $-4x^2 + 16x - 16 = 0$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2 = 0$$

$$x - 2 = 0$$

$$x = 2$$

$$S = \{2\}$$

d) $2x^2 - 16 = 0$

$$x^2 - 8 = 0$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

$$x_1 = 2\sqrt{2}$$

$$x_2 = -2\sqrt{2}$$

$$S = \{-2\sqrt{2}; 2\sqrt{2}\}$$

e) $2x^2 - 4x - 7 = 0$

$$\Delta = b^2 - 4ac = 16 + 56 = 72$$

$$x = \frac{4 \pm \sqrt{72}}{4} = \frac{4 \pm 6\sqrt{2}}{4}$$

$$x_1 = \frac{2 + 3\sqrt{2}}{2}$$

$$x_2 = \frac{2 - 3\sqrt{2}}{2}$$

$$S = \left\{ \frac{2 - 3\sqrt{2}}{2}; \frac{2 + 3\sqrt{2}}{2} \right\}$$

Exercice 2.2.6

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $(x - 1)(x + 1) = (x - 2)^2$

$$x^2 - 1 = x^2 - 4x + 4$$

$$4x = 4 + 1$$

$$4x = 5$$

$$x = \frac{5}{4}$$

$$S = \left\{ \frac{5}{4} \right\}$$

b) $(y - 3)(y + 4) = 0$

$$y - 3 = 0 \quad y_1 = 3$$

$$y + 4 = 0 \quad y_2 = -4$$

$$S = \{-4; 3\}$$

c) $2x^2 + 2 = 2(x^2 + 2)$

$$2x^2 + 2 = 2x^2 + 4$$

$$2 = 4$$

$$S = \emptyset$$

d) $x^2 - 9 = 0$

$$x^2 = 9$$

$$x = \pm 3$$

$$S = \{-3; 3\}$$

e) $10x^2 + 10 + 40x = 60 + 10x^2$

$$10 + 40x = 60$$

$$40x = 50$$

$$x = \frac{5}{4}$$

$$S = \left\{ \frac{5}{4} \right\}$$

f) $x^2 + 5x = x^2$

$$5x = 0$$

$$x = 0$$

$$S = \{0\}$$

Exercice 2.2.7

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $2x^2 = 2(x + 4)^2$

$$x^2 = (x + 4)^2$$

$$x^2 = x^2 + 8x + 16$$

$$0 = 8x + 16$$

$$-8x = 16$$

$$x = -2$$

$$S = \{-2\}$$

b) $(x + 1)^2 = x^2 + 2x + 1$

$$x^2 + 2x + 1 = x^2 + 2x + 1$$

$$x = \mathbb{R}$$

$$S = \mathbb{R}$$

c) $5x^2 - 10x + 5 = 0$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x - 1 = 0$$

$$x = 1$$

$$S = \{1\}$$

d) $\frac{x^2}{2} + 3x = \frac{x^2}{2} + 1$

$$3x = 1 \quad x = \frac{1}{3}$$

$$S = \left\{\frac{1}{3}\right\}$$

e) $0 = 3(x - 7)^2 - 147$

$$3(x - 7)^2 - 147 = 0$$

$$3(x^2 - 14x + 49) - 147 = 0$$

$$3x^2 - 42x + 147 - 147 = 0$$

$$3x^2 - 42x = 0$$

$$3x(x - 14) = 0$$

$$3x = 0 \quad x_1 = 0$$

$$x - 14 = 0 \quad x_2 = 14$$

$$S = \{0; 14\}$$

Exercice 2.2.8

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $3(x - 1) + (x - 1)(x - 2) + 2 = 2x$

$$3x - 3 + x^2 - 2x - x + 2 + 2 = 2x$$

$$1 + x^2 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x = 1$$

$$S = \{1\}$$

b) $x(x^2 - 5x + 4) = 0$

Lorsque le produit des facteurs est égal à 0, au moins un des facteurs est 0

$$x = 0$$

$$x^2 - 5x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

$$x = 1$$

$$x = 4$$

$$S = \{0; 1; 4\}$$

c) $9x^2 + 18x + 9 = 4x(-3x - 3)$

$$9x^2 + 18x + 9 = -12x^2 - 12x$$

$$9x^2 + 18x + 9 + 12x^2 + 12x = 0$$

$$21x^2 + 30x + 9 = 0$$

$$7x^2 + 10x + 3 = 0$$

$$\Delta = 100 - 84 = 16$$

$$x = \frac{-10 \pm 4}{14}$$

$$x_1 = -\frac{6}{14} = -\frac{3}{7}$$

$$x_2 = -\frac{14}{14} = -1$$

$$S = \{-\frac{3}{7}; -1\}$$

Exercice 2.2.9

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $(x + 7)^2 = (4x + 3)^2$

$$x^2 + 14x + 49 = 16x^2 + 24x + 9$$

$$x^2 + 14x + 49 - 16x^2 - 24x - 9 = 0$$

$$-15x^2 - 10x + 40 = 0$$

$$3x^2 + 2x - 8 = 0$$

$$\Delta = 4 + 96 = 100$$

$$x = \frac{-2 \pm 10}{6}$$

$$x_1 = -\frac{12}{6} = -2$$

$$x_2 = \frac{8}{6} = \frac{4}{3}$$

$$S = \left\{ -2 ; \frac{4}{3} \right\}$$

b) $(2x + 3)(5x - 4) - (5x - 4)(-3x + 1) = 0$

$$10x^2 - 8x + 15x - 12 - (-15x^2 + 5x + 12x - 4) = 0$$

$$10x^2 - 8x + 15x - 12 + 15x^2 - 5x - 12x + 4 = 0$$

$$25x^2 - 10x - 8 = 0$$

$$\Delta = b^2 - 4ac = 100 + 800 = 900$$

$$x = \frac{10 \pm \sqrt{900}}{50} = 10 \pm 30/50$$

$$x_1 = \frac{10 + 30}{50} = \frac{4}{5} \quad x_2 = \frac{10 - 30}{50} = -\frac{2}{5}$$

$$S = \left\{ -\frac{2}{5} ; \frac{4}{5} \right\}$$

Exercice 2.2.10

Détermine l'ensemble des solutions de chacune des équations suivantes.

a) $3(2x - 2) = 4x + 7$

$$6x - 6 = 4x + 7$$

$$2x = 13$$

$$x = \frac{13}{2}$$

$$S = \left\{ \frac{13}{2} \right\}$$

b) $7x^5 - 7x^4 - 84x^3 = 0$

$$7x^3(x^2 - x - 12) = 0$$

$$7x^3(x + 3)(x - 4) = 0$$

$$x_1 = 0 \quad x_2 = -3 \quad x_3 = 4$$

$$S = \{-3; 0; 4\}$$